DEPENDENCE OF EFFECTIVE LINEAR ATTENUATION COEFFICIENT ON X-RAY TUBE VOLTAGE RIPPLE

T. Porubszky, Medicor, Budapest, Hungary

The so-called non-invasive X-ray tube voltage measuring devices generally use signals of two detectors having copper filters of different thicknesses. Although these devices are conventionally called peak kilovoltage (kVp) meters, influence of tube voltage and current waveforms on the measurement results need some considerations. One can choose only the calibrations corresponding to the conventional rectifying systems even on devices of the highest level, some other types provide only two calibrations: one for single-phase and another for three-phase waveforms. Although in the USA there is developed a device capable for measurements of medium frequency X-ray generators (Victoreen NERO-C), however, it is a very expensive top device. It seems to be unsolved all over the world the daily non-invasive X-ray tube voltage measurement of medium frequency X-ray generators. Considering that most of these generators are monotank types where voltage divider methods are not applicable, it can be seen that solution of this problem is very urgent.

Theoretical bases

One of the physical quantities used for characterizing radiation quality is the so-called effective linear attenuation coefficient. It can be defined from the following expression:

$$\psi = \int_0^{E\max} \psi_0(E) \cdot e^{-\mu(E)x} \cdot \mathrm{d}E = \psi_0 \cdot e^{-\bar{\mu}x} \tag{1}$$

where ψ is the energy fluence rate (or with older terminology "intensity") of the X-ray beam having originally a ψ_0 energy fluence rate, passed through an absorber having a thickness x, E is the photon energy, μ is the linear attenuation coefficient of the absorber. $\psi_0(E)$ means spectral distribution, $\bar{\mu}$ can be expressed from this formula:

$$\bar{\mu} = -\frac{1}{x} \ln \frac{\psi}{\psi_0} \tag{2}$$

It can be seen that value of μ depends upon not only the kind of the absorber but on its thickness *x*, too. Therefore $\bar{\mu}$ can not be considered as a characteristic of the radiation but that of the interaction of the absorber of the given thickness with the radiation. Its value can be determined with the aid of detectors of energy integrating mode. Detectors of non-invasive X-ray tube voltage measuring devices can be considered to be such types in a good approximation.

Our goal was to calculate the dependence of $\bar{\mu}$ on tube voltage pulsation for estimating the attainable accuracy of non-invasive X-ray tube voltage measuring methods. Accuracy of determining peak kilovoltage by non-invasive methods is limited as only such differences of kilovoltage can be distinguished that cause higher differences in $\bar{\mu}$ than the maximum ripple differences for the same peak kilovoltage.

Formulas

The calculation is based on formulas given in an earlier paper. Taking into account some known expressions (i.e. mass attenuation coefficients of mixtures, absorbed dose from a polyenergetic beam, averaging of time dependent quantities) we have obtained the following formula for the absorbed dose from narrow X-ray beams generated by pulsating potential X-ray generators having any waveform, passed through any attenuating medium:

$$D(x)=\frac{t}{Tx^2}\int_0^{E_{max}} A(E)\cdot\left(\frac{\mu_{en}}{\rho}\right)(E,x)\cdot E\cdot B(E)\cdot \mathrm{d}E \qquad (3)$$

where

$$A(E)=\exp\left\{-\int_{x_0}^{x}\left[\sum_{i=1}^{n(x')} w_i(x')\cdot\left(\frac{\mu}{\rho}\right)_i(E)\right]\cdot\rho(x')\cdot \mathrm{d}x'\right\} \qquad (4)$$

and

$$B(E)=\int_0^T \varphi_0\big(U(t'),E\big)\cdot i(t')\cdot \mathrm{d}t' \qquad (5)$$

where x is the space coordinate, $x=0$ is the point of origin of the X-ray beam (tube focus), x_0 is the position of the tube housing window, t' is the time, t is the duration of the irradiation, T is the period time of the tube voltage*, E is the photon energy, φ_0 *is* the spectral distribution of (photon) fluence rate with respect to energy normalized to unit i and x, ρ is the density, μ/ρ is the mass attenuation coefficient, μ_{en}/ρ is the mass energy absorption coefficient and D is the absorbed dose. The quantity in square brackets represents the mass attenuation coefficient of an n-component compound or mixture in which w_i is the fraction by weight of the i-th component and $(\mu/\rho)_i$ is its mass attenuation coefficient, $A(E)$ expresses the total transmitted fraction of photons of energy E of the beam passed through all the attenuating media while $B(E)$ represents the average fluence rate spectrum normalized to unit distance from the tube focus without attenuation. In the formulas all variables are written as arguments. The inherent filtration of the X-ray tube is included in the φ_0 spectrum while added filters are to be taken into account among the attenuating media.

Based on the former expression, the fluence can be written in the following form:

$$\Phi(x)=\frac{t}{T\cdot x^2}\int_0^{E_{max}} A(E)\cdot B(E)\cdot \mathrm{d}E \qquad (6)$$

while the energy fluence is

$$\Psi(x)=\frac{t}{T\cdot x^2}\int_0^{E_{max}} A(E)\cdot E\cdot B(E)\cdot \mathrm{d}E \qquad (7)$$

(These equations do not include the contribution of the scattered radiation – narrow-beam geometry – as this contribution can not be expressed in such a direct form but can be estimated using some approximation methods.)

Taking expressions (1) and (7) into consideration, $\bar{\mu}$ can be calculated for any $U(t)$ and $i(t)$ waveforms and x copper filter thicknesses.

---

* i is the tube current, U is the tube voltage

Results

A computer program has been written for the calculation of $\bar{\mu}$. In table 1 can be seen the *U(t)* and *i(t)* idealized (analytical) waveforms which were used for calculations of the given generator types. As the application of the listed waveforms was not enough for conclusions, calculations were made with some "deformed" 6-pulse waveforms, too. It means that sine waves compressed along the t-axis are superimposed but six ones corresponding to the original period time. Therefore the time interval for calculation is unchanged (i.e. (0, *T*/12)), but using the identity

$$\sin\, 2\pi\left(\frac{t}{T}+\frac{1}{6}\right)\equiv \cos\, 2\pi\left(\frac{t}{T}-\frac{1}{12}\right)$$

a k-fold compression

$$\cos\, 2\pi k\left(\frac{t}{T}-\frac{1}{12}\right)$$

is made which for *k*=1 naturally gives the original values. As for sine function one twelfth of the period is $\pi/6$, therefore using

$$\cos\, k\,\frac{\pi}{6}=a$$

the values of k were determined for given

$$a=\frac{U_{\min}}{U_{\max}}$$

values.

Table 1: Analytical approximations of waveforms

Symbol	Type	Tube voltage	Tube current	Time interval
0	2 pulse	$U(t)=U_{\max}\cdot\sin\,\frac{2\pi t}{T}$ but $U(t)\geq 26\,\text{kV}$	$i(t)\sim U^{a}(t)$	$0\leq t\leq\frac{T}{4}$
1	6 pulse	$U(t)=U_{\max}\cdot\sin 2\pi\left(\frac{t}{T}+\frac{1}{6}\right)$	$i(t)=\frac{i_{\max}}{2}\cdot\left[1+\sin 2\pi\cdot\left(\frac{2t}{T}+\frac{1}{12}\right)\right]$	$0\leq t\leq\frac{T}{12}$
2	12 pulse	$U(t)=U_{\max}\cdot\cos\,\frac{2\pi t}{T}$	$i(t)=\frac{i_{\max}}{2}\cdot\left(1+\cos\frac{4\pi t}{T}\right)$	$0\leq t\leq\frac{T}{24}$
3	40% spike			$0\leq t\leq\frac{T}{4}$
4	20% spike	$U(t)=U_{\max}\cdot\left(C_1\cdot\frac{t}{T}+C_2\right)$	$i(t)=i_{\max}\left(C_3\cdot\frac{t}{T}+C_4\right)$	
5	10% spike	$U(t)=U_{\max}$	$i(t)=i_{\max}$	
6	5% spike			$\frac{T}{4}\leq t\leq T$
7	Constant	$U(t)=U_{\max}$	$i(t)=i_{\max}$	any

T is the period time. Waveforms corresponding to the given time intervals are repeated integer times within *T*. Thus there is enough to calculate for one interval. Values of the constants:

Type 3	C_1=1,6	C_2=0,6	C_3=1,2	C_4=0,7	Type 4	C_1=0,8	C_2=0,8	C_3=0,6	C_4=0,85
Type 5	C_1=0,4	C_2=0,9	C_3=0,3	C_4=0,925	Type 6	C_1=0,2	C_2=0,95	C_3=0,15	C_4=0,9625

Figure 1 shows the used data where

$$R = \frac{U_{\max} - U_{\min}}{U_{\max}} \tag{8}$$

is the so-called percentage ripple. Calculations were performed in the so-called characteristic tube current approximation, i.e. tube currant is taken as a power function of tube voltage.

For G=3, 4, 5 and 6 (medium frequency) generator type waveforms the R pulsation was determined differently from the former standard formula. As in these cases tube current and voltage are constant in *75%* of the period tine and pulsate only in *25%* of it, for these waveforms 25% of the value given by the standard formula was considered to be the percentage ripple, for these waveforms.

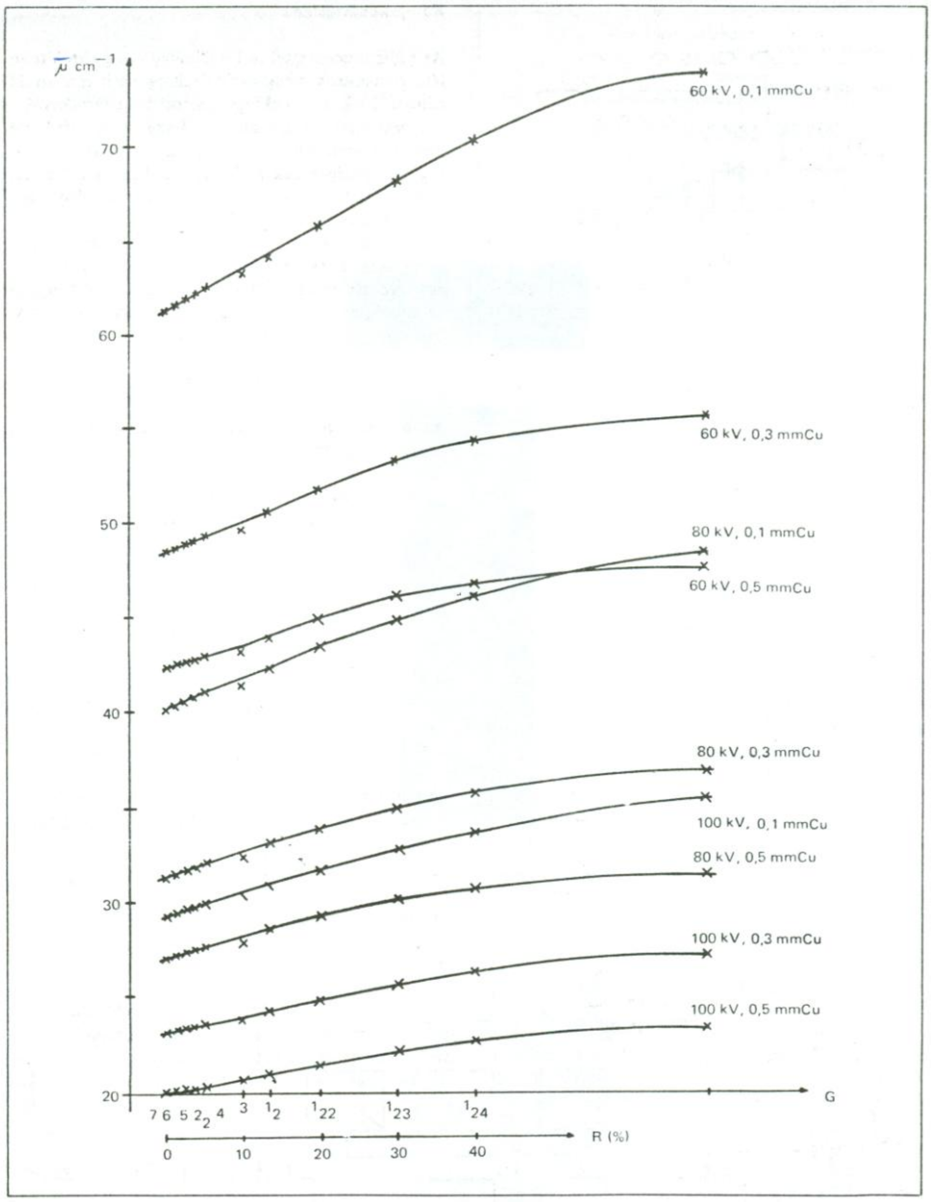

Figure 1.

Dependence of effective linear attenuation coefficient on voltage ripple. On the horizontal axis: percentage ripple $\left(\frac{U_{max} - U_{min}}{U_{max}}\right)$*, corrected to the period time, and generator types (7: DC, 0: 2-pulse, 2: 12- pulse, 1: 6-pulse,* 1_{22}*,* 1_{23}*,* 1_{24} *: 6-pulse waveforms, with increased pulsation of 20, 30 and 40 %, respectively, 3, 4, 5, 6: waveforms modelling inverter generators), on the vertical axis: the effective linear attenuation coefficient (in units 1/cm), X-ray generator voltage and absorber thickness are parameters*

Conclusions

A research work for constructing a non-invasive X-ray tube voltage measuring device is being made in VNIIIMT** Moscow, with the active cooperation of X-ray Division of Medicor Budapest. This work was a part of this cooperation. An experimental confirmation was made for some pulsation values, and measured results are identical with calculated ones within the measuring accuracy. (However, it is different to compare more experimental values as the degree of pulsation can be influenced only slightly in most generator types.) Practical construction is being made by colleagues of the VNIIIMT A. I. Leitchenko and T. V. Danilenko.

An essential conclusion can be drawn from the figure: for kVp-measuring within 5-6% accuracy two calibrations are sufficient one of them for 1- and 2-pulse generators and the other for all other types (i.e. constant potential, medium frequency, 6- and 12-pulse).

Reference

T. Porubszky: Phys. Med. Biol. 31, 371-381 (1986).

*** VNIIIMT = All-Union Scientific Research Institute for Medical Engineering*